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Turbulence Characteristics of Liquids in Pipe Flow

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The turbulence characteristics of a fluid stream in a given duct geometry are of interest in a wide variety of applications. For example, the dispersion of mass and the transfer of heat by turbulent diffusion, problems of particular importance to chemical engineers, are greatly influenced by these turbulence characteristics. Typical parameters used to describe turbulence are the relative intensity and the integral scales of turbulence. These parameters may be obtained from either Lagrangian (observer moves with the fluid) or Eulerian (observer is stationary) methods. The mixing properties of a turbulent fluid are closely related to, and may be predicted from, Lagrangian turbulence parameters. Unfortunately there is, at present, no instrumentation available to measure these quantities directly. Eulerian turbulence parameters, however, can be measured directly with hot-wire or hot-film anemometry. A relation between the Eulerian and Lagrangian turbulence parameters would be very useful even it if was empirical.

Most available data on the relative intensity and integral scales of turbulence in pipe flow appear to have been obtained for air systems. It is considerably more difficult, however, to obtain these data for liquids, particularly ordinary tap water or sea water (*1*). The local physical and electrical properties of these liquids may vary with time and location and may differ considerably from the average fluid properties. These variations lead to erratic readings; consequently, great difficulty may be experienced in obtaining accurate, reproducible values of absolute liquid flow rates with hot-film or wire anemometry techniques. Nevertheless, values of the relative intensity of turbulence, correlation coefficients, and integral scales of turbulence were obtained in this investigation which were directly comparable to those in air. It is believed that this agreement was obtained because all the above parameters are relative values and thus self-compensating.

Further, almost no data on Eulerian integral scales of turbulence have been reported in the Reynolds number range below 200,000. A probable reason for this paucity of data is that low frequency ($<$ 5 cycles/sec.) components in the turbulent signal are encountered, and most conventional electronic instrumentation fails in this region.

This investigation reports data on relative intensity and Eulerian scales of turbulence at the center of a pipe through which water was pumped with velocities ranging from 0.5 to 4.0 ft./sec. (N_{Re} = 19,000 to 160,000). The data are compared with those reported with air as the fluid. Results are correlated, insofar as possible, with the Reynolds number of flow and with eddy diffusivities and Lagrangian turbulence parameters reported by other investigators.

THEORY

Einstein (*2*) showed that the mean square displacement of a particle in Brownian movement, after a long interval of time, may be represented by the equation

$$\overline{Y^2} = [2t]\left[\,0.57\sqrt{\overline{C^2}}\,l\,\right] \tag{1}$$

For molecular diffusion $\left[\,0.57\sqrt{\overline{C^2}}\,l\,\right]$ may be replaced by a molecular diffusion coefficient D_{mol} to give

$$\overline{Y^2} = [2t]\,[D_{mol}]$$

Taylor (*3, 4, 5*) showed that if a number of particles having properties similar to the fluid were released in a field of uniform turbulence, the mean value of the displacement of a particle is given by

$$\overline{Y^2} = [2t]\left[\,\overline{C^2}\int_0^\infty R_L(t')\,dt'\,\right] \tag{3}$$

where $R_L(t') = \overline{C_tC_{t+t'}}/\overline{C_t^2}$ is the Lagrangian correlation coefficient between the velocity of a particle at times t and $t + t'$. If one defines a Lagrangian integral time scale of turbulence $T_L = \int_0^\infty R_L(t')dt'$, Equation (3) becomes

$$\overline{Y^2} = [2t]\,[\overline{C^2}\,T_L] \tag{4}$$

which may be rearranged to give

$$\overline{Y^2} = [2t]\left[\sqrt{\overline{C^2}}\left\{\sqrt{\overline{C^2}}\,T_L\right\}\right] \tag{5}$$

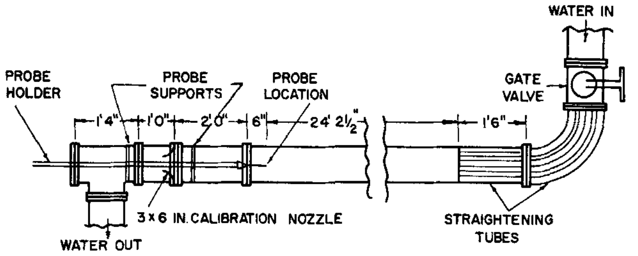

Fig. 1. Detailed diagram of test section.

Comparing Equations (1) and (5) one may define, analogous to the mean free path l, a Lagrangian integral length scale of turbulence $\Lambda_L = \left[\sqrt{\overline{C^2}}\, T_L\right]$ which reduces Equation (5) to the form

$$\overline{Y^2} = [2t]\left[\sqrt{\overline{C^2}}\,\Lambda_L\right] \qquad (6)$$

Finally, an eddy diffusivity D_E, analogous to the molecular diffusivity D_{mol}, may be defined by $D_E = \left[\Lambda_L \sqrt{\overline{C^2}}\right]$ to give

$$\overline{Y^2} = [2t]\,[D_E] \qquad (7)$$

Experimentally, it is often more convenient to use an Eulerian coordinate system because the turbulent velocity at a point may be measured relatively easily with hot-wire or hot-film anemometry techniques. Analogous to the Lagrangian coordinate system, one may therefore define the following terms

Eulerian correlation coefficient $R_E(t')$ where

$$R_E(t') = \overline{u_t u_{t+t'}}/\overline{u_t^2} \qquad (8)$$

Eulerian integral time scale of turbulence T_E

$$\text{where} \quad T_E = \int_0^\infty R_E(t')\,dt' \qquad (9)$$

Eulerian integral length scale of turbulence Λ_E

$$\text{where} \quad \Lambda_E = (\overline{U})\,(T_E) \qquad (10)$$

for homogeneous, isotropic turbulence as exists at the center of a pipe (6).

EXPERIMENTAL

A schematic diagram of the test section is shown in Figure 1. A 26.2-ft. section of straight 6-in. pipe was introduced before the measuring device to help insure that a completely established velocity profile was obtained. The 26.2-ft. section of pipe was followed by a 2.0-ft. section of pipe, a 3- × 6-in. nozzle, a flexible coupling, and a 6- × 6-in. tee. The water entered one arm of the tee and left at the stem. The other arm of the tee was fitted with a steel plate in which was embedded a cylindrical brass bearing. The hot-film probe was held in a 5.0 ft., stainless steel tube (O.D. = ⅞ in., I.D. = ¼ in.) which could slide through the brass bearing. The probe holder was also supported by two other bearings situated 1.3 and 2.5 ft. away. The probe was calibrated by centering it in the throat of the nozzle at known flow rates. Immediately after calibration, the probe was pushed out of the nozzle into the end of the 26.2-ft. section of pipe for turbulence measurements.

The purpose of the 2.0-ft. section of pipe was twofold. It decreased any effects the nozzle might have on the turbulence in the end of the 26.2-ft. section of pipe. It also was fitted with a removable half section of pipe so that the probe could be cleaned with acid and water before every run.

Since there was some doubt whether a fully developed velocity profile would be obtained with only 50 pipe diameters of undisturbed pipe section, straightening tubes were introduced in the inlet section of the pipe for one run. The straightening tubes were composed of a bundle of 18-in. straight, stainless steel tubes (O.D. = ⅞ in., I.D. = ¾ in.) inserted in the straight section of the pipe and a bundle of plastic hose (O.D. = 1 in., I.D. = ⅝ in.) inserted in the long elbow. The straightening tubes were held in place, on the downstream side, by a ¼-in. mesh screen. It was hoped that the net result of these additions would be to break up possible eddies formed by the valve and long elbow to such a small size that they would decay long before reaching the point at which the turbulence measurements were made.

A hot-film anemometer (7) was used to obtain an electronic signal corresponding to the turbulent, fluctuating component of velocity. This signal was led into a true root-mean-square voltmeter to obtain the intensity of turbulence. The signal was simultaneously displayed on an oscilloscope screen, photographed, and then analyzed numerically to obtain the correlation coefficients and the integral scales of turbulence.

The relative intensity of turbulence was obtained as follows. The slope of the anemometer calibration curve was first obtained giving the number of volts corresponding to unit change in velocity. The reading on the true root-mean-square voltmeter was multiplied by this slope to get the intensity of turbulence which was then divided by the fluid velocity at the center of the pipe to give the relative intensity of turbulence. Owing to the drift of the hot-film anemometer and possibly a change in the physical properties of the water, the slope of the calibration curve changed during each run. The following procedure was therefore adopted in an attempt to compensate for this drift. Calibration curves were obtained before and after each run, and their slopes were calculated. The difference in the two slopes was assumed to be a linear function of time to allow for a correction to be applied to the calibration slope at each point in the run. Further, the velocities were increased (or decreased) during the first half of each run and then decreased (or increased) during the second half of each run in an effort to average out any definite bias in the changes of the calibration slopes.

Eulerian correlation coefficients $R_E(t')$ were obtained with the following procedure. Seventeen oscilloscope pictures were picked for good pictorial quality (but otherwise randomly selected) for each mean velocity through the pipe. Ninety values of the ordinate were obtained from each photograph at equal intervals of 0.1 in. on the abscissa. (The corresponding time interval ranged from 0.004 to 0.040 sec. This interval depended upon the oscilloscope sweep speed which was picked arbitrarily to give clear, legible oscilloscope traces.) The numerical values of the ordinate were punched into I.B.M. cards and average values $\overline{u_t}$, $\overline{u_t^2}$, and $\overline{u_t u_{t+t'}}$ were calculated for each set of ninety values. Overall averages for all seventeen sets were then determined and used in Equation (8) to obtain the Eulerian correlation coefficient $R_E(t')$.

It was necessary to ensure that the data obtained from the seventeen oscilloscope pictures was an adequate sample of the total turbulent trace and that they would both have the same statistical characteristics. The data from each of the pictures were scanned to determine in which picture $\overline{u_t^2}$ deviated most

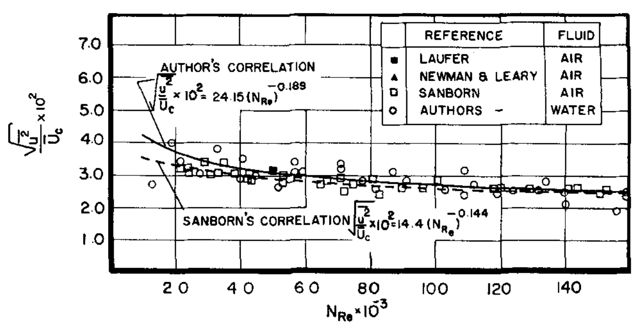

Fig. 2. Relative intensity as a function of Reynolds number.

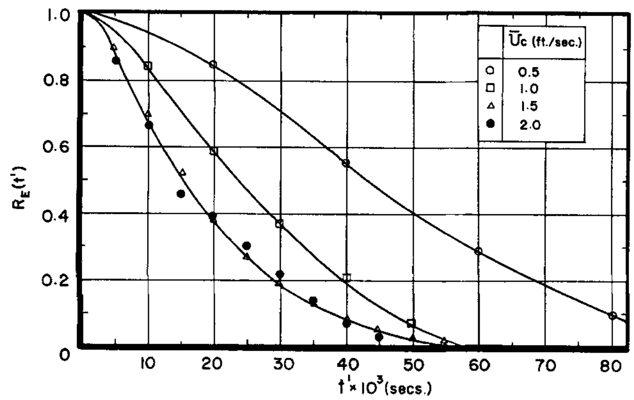

Fig. 3. Eulerian time correlation coefficients as a function of time delay (velocities: 0.5 to 2.0 ft./sec.).

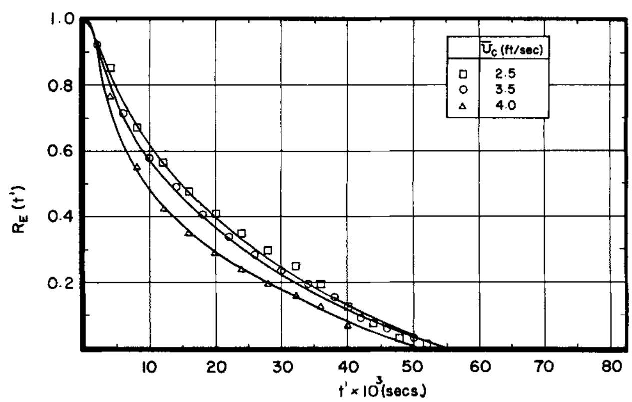

Fig. 4. Eulerian time correlation coefficients as a function of time delay (velocities: 2.5 to 4.0 ft./sec.).

widely from the value of $\overline{u_t^2}$ calculated from all seventeen pictures. The correlation coefficients were then recalculated with the data from this picture twice, and therefore with essentially eighteen sets of data. In no case was the correlation coefficient found to change by more than ± 0.02. Since the additional data included represented the most adverse data which could be used, it was assumed that the seventeen oscilloscope pictures gave a sufficient sample of the turbulent trace. Further, it was later found that the time sample of the turbulent trace used was always greater than 150 times the Eulerian integral time scale of turbulence. This would tend to support the contention that an adequate sampling was obtained.

The Eulerian correlation coefficients obtained were then plotted against the time delay. For consistency in results, the Eulerian integral time scales of turbulence were calculated by only considering the area under the $R_E(t')$ curve for $R_E(t') > 0$. Hence the Eulerian integral time scales of turbulence T_E obtained in this work actually correspond to their definition by the equation

$$T_E = \int_0^{t_n'} R_E(t')dt'$$

where t_n' is the value of the time delay at which the Eulerian autocorrelation coefficient $R_E(t')$ first becomes zero.

VALIDITY OF DATA AND COMPARISON WITH THOSE OF OTHER INVESTIGATORS

The relative intensity of turbulence data obtained could be divided into two sets, each set containing two groups: (1a). without straightening tubes and Reynolds number increasing, (1b). without straightening tubes and Reynolds number decreasing; (2a). with straightening tubes and Reynolds number increasing, (2b). with straightening tubes and Reynolds number decreasing. To improve the reliability of interpreting the somewhat scattered data, a statistical analysis was made, following the methods outlined in reference 8.

Each group of data was tested against the other three groups for differences between the slopes and between the adjusted means. No significant difference at the 95% confidence level was noted between the slopes of any two groups of data. When the adjusted means were tested against each other, significant differences were obtained only when group (2b) was compared with group (1b), and also when group (2b) was compared with group (2a). Further, the difference between (2b) and (2a) was more significant than the difference between (2b) and (1b).

The results of this analysis were interpreted as follows. The introduction of the straightening tubes made no real difference in the relative intensity of turbulence data, although two apparently significant differences were found in comparing data groups. The largest difference was found comparing group (2a) with (2b). This suggests the method of correcting for anemometer drift was only partially successful.

Even though the above analysis suggests the data from group (2b) do not belong to the same population as the other data, they were nevertheless included in the overall correlation since it was felt that the bias introduced by the anemometer drift was largely compensated by the opposite bias introduced in set (2a). With a least squares fit it was found that all the data could be represented by the equation

$$\sqrt{\overline{u^2}}/\bar{U}_c \times 10^2 = (24.15)\ (N_{Re})^{-0.189} \qquad (11)$$

Further, the 95% confidence range of the relative intensity of turbulence (at $N_{Re} = 70{,}640$) was found to be 0.0291 ± 0.0013. These data and the curve representing Equation (11) are shown in Figure 2, together with literature values to be discussed below.

Relative intensity of turbulence data, in the same Reynolds number range as those obtained by this investigator, were also obtained by Sanborn (*9*) using both constant current and constant temperature hot-wire anemometers. Isolated points have also been reported by Newman and Leary (*10*) and Laufer (*11, 12*). All these data, obtained with air as the fluid, are shown in Figure 2 to be in substantial agreement with data obtained in this investigation. Measurements by Skillern (*13*) gave relative intensity values of 0.03, but as an undisturbed pipe section of only 12 pipe diameters was used, these measurements were not considered reliable. Relative intensity measurements by Flint, Kada, and Hanratty (*14*) in the same Reynolds number range, using water as the system, are much higher. Owing to the fact that these measurements were obtained indirectly from diffusion measurements and are therefore a Lagrangian relative intensity $\sqrt{\overline{C^2}}/\bar{U}_c$, and because of their inconsistency with the Eulerian data of other investigators, these results will not be included in the correlation proposed by these investigators. At higher Reynolds numbers (> 200,000) Baldwin and Walsh (*6*) report constant values of 0.035 for the relative intensity. Mickelsen (*15*) reports values averaging at about 0.031, and Laufer has one value of 0.030 at a Reynolds number of 500,000. If these data are plotted with the data obtained at the lower Reynolds number, a discontinuity is noted in the relative intensity of turbulence curve in the Reynolds number range 150,000 to 200,000. However, the accuracy of relative intensity of turbulence data is highly dependent on instrumentation and the methods of calibration used for the hot-wire anemometer equipment, and it is felt that additional experimental work in this range would be necessary to substantiate the presence or absence of a discontinuity.

Eulerian autocorrelation coefficients as a function of the time delay are shown in Figures 3, 4, and 5. In all cases (except for the run at a center-line velocity = 3.0 ft./sec.) the correlation coefficients actually decreased to small negative values before attaining steady values of zero. In the run at center-line velocity = 3.0 ft./sec. (Figure 5) the autocorrelation coefficient exhibited interesting behavior. It never attained a zero value, and there was a regular rise and fall superimposed on the exponential decay of the $R_E(t')$ curve. It is speculated that an environmental off-set sinusoidal 60 cycles/sec. signal was recorded superimposed on the turbulent signal. This signal would cause regular maxima to occur in the $R_E(t')$ curve at intervals of 0.016 sec., which corresponds closely to the experimental curve obtained. This run was discarded for correlation purposes.

The Eulerian integral length scale of turbulence, normalized by dividing by the diameter of the pipe, is shown as a function of the Reynolds number in Figure 6. A least-squares fit of these data is represented by the equation

$$[\Lambda_E/2a] \times 10^2 = (0.846)\ (N_{Re} \times 10^{-3})^{0.509}$$

The 95% confidence range of $\Lambda_E/2a$ (at Reynolds number = 69,700) was found to be 0.073 ± 0.013.

There appears to be very little published data on Eulerian integral scales of turbulence in this Reynolds number range. Sanborn (*9*) had reported spectral energy measurements at a Reynolds number of 25,000. These measurements were used to calculate the Eulerian integral scale of turbulence, and this single point agrees reasonably well with the values obtained in this study as indicated in Figure 6. Eulerian integral length scales of turbulence at higher Reynolds numbers (200,000) have been obtained by Baldwin and Walsh (*6*), Mickelsen (*15*), and Laufer (*11, 12*). If the ratio $\Lambda_E/2a$ is plotted against the Reynolds number, the data obtained by Baldwin and Walsh, by Laufer, and those of this investigation appear to follow the same trend, but Mickelsen's results are lower. Since Mickelsen's data were obtained with a calming section of only 35 pipe diameters, it was felt that his results are less reliable and they are therefore not included in the correlation proposed later by the authors.

RELATION BETWEEN EULERIAN AND LAGRANGIAN TURBULENCE PARAMETERS

Taylor (*16*) showed that for the flow of a fluid through pipes at higher Reynolds numbers, the eddy diffusivity D_E is directly proportional to the diameter of the pipe $2a$ and to the friction velocity u^*; that is

$$D_E = (\text{constant})\ (2a)\ (u^*) \qquad (12)$$

This equation may be combined with the definition of the

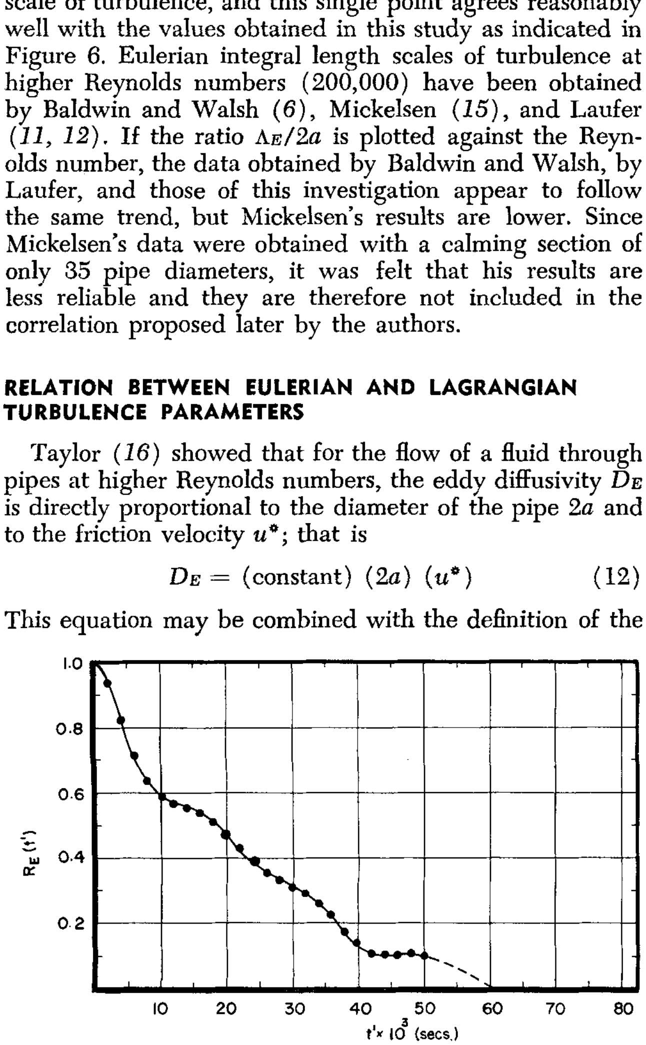

Fig. 5. Eulerian time correlation coefficient as a function of time delay (velocity: 3.0 ft./sec.).

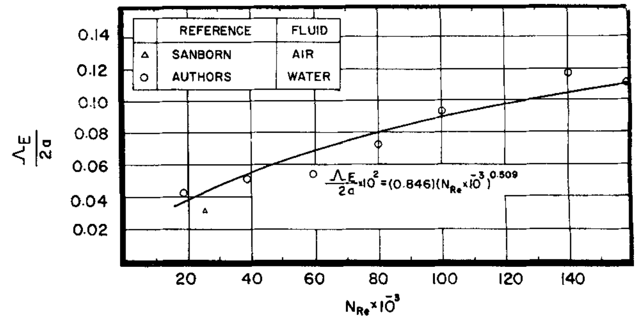

Fig. 6. Ratio of Eulerian integral length scale of turbulence to diameter of pipe as a function of Reynolds number.

eddy diffusivity

$$D_E = (\Lambda_L)\left(\sqrt{\overline{C^2}}\right)$$

to give

$$[\Lambda_L]\left[\sqrt{\overline{C^2}}\right] = (\text{constant})\ (2a)\ (u^*)$$

which may be further rearranged to give

$$\left[\frac{\Lambda_L}{2a}\right] \frac{\sqrt{\overline{C^2}}}{\overline{U}_c} \Big/ \left[\frac{u^*}{\overline{U}_{avg}}\right] =$$

$$\left[\frac{\Lambda_L}{2a}\right] \frac{\sqrt{\overline{C^2}}}{\overline{U}_c} \Big/ \left[\sqrt{\frac{f}{2}}\right] = \text{constant} \qquad (13)$$

Equation (13)shows that the group

$$\left[\frac{\Lambda_L}{2a}\right] \frac{\sqrt{\overline{C^2}}}{\overline{U}_c} \Big/ \left[\sqrt{\frac{f}{2}}\right]$$

which may be termed the *Lagrangian parameter* is a constant, independent of the Reynolds number. This relationship was developed by Taylor for the highly turbulent region where the universal velocity profile and the ratio $\overline{U}_{avg}/\overline{U}_c$ are both independent of the Reynolds number. It would appear logical to try to correlate the Lagrangian turbulence characteristics at lower Reynolds number by also plotting the Lagrangian parameter against the Reynolds number. It is interesting to note that Flint, Kada, and Hanratty (*14*), using purely dimensional reasoning, used the group $(D_E)/(2a)\ (\overline{U}_c)$ to correlate their Lagrangian data; this group, of course, is the numerator of the Lagrangian parameter. Further, if one assumes that some relation exists between the Lagrangian and Eulerian parameters (even though derivation of this relationship may be impractical), it would appear reasonable to try to correlate the Eulerian turbulence data by plotting the group

$$\left[\frac{\Lambda_E}{2a}\right] \frac{\sqrt{\overline{u^2}}}{\overline{U}_c} \Big/ \sqrt{\frac{f}{2}}$$

(for convenience called the *Eulerian parameter*) against the Reynolds number. The Lagrangian and Eulerian parameters are shown as functions of the Reynolds number in Figure 7.

The Lagrangian parameter, calculated from the smoothed data of Hanratty et al. (*14*), appears to decrease slightly and then attain a constant value of 0.016. With the data of Baldwin and Walsh (*5*) at higher Reynolds number (200,000) the Lagrangian parameter is still independent of the Reynolds number but appears to have

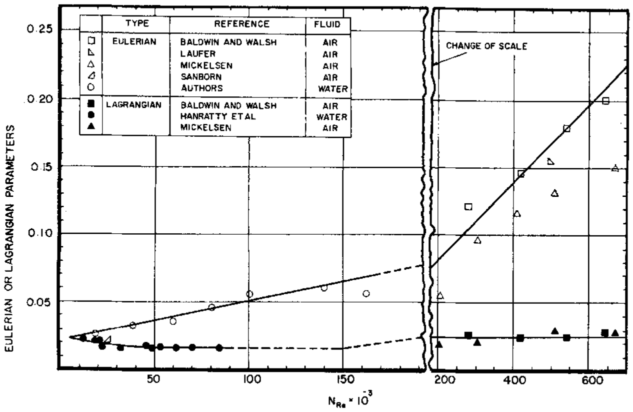

Fig. 7. Eulerian and Lagrangian parameters as a function of Reynolds number.

a slightly higher value of 0.025. The difference in the two values is slight but quite perceptible. Since a possible discontinuity also occurs in the relative intensity of turbulence data in this range, the authors feel that the presence of a discontinuity in the fluid flow characteristics should not be entirely discounted.

The Eulerian parameter, calculated primarily from the data of Baldwin and Walsh and the data obtained in this work, was observed to increase steadily with the Reynolds number. Even though the possibility of a discontinuity does exist, the results (excluding Mickelsen's work) were correlated by a straight line:

Eulerian parameter $= 0.0214 + (0.0294)\ (N_{Re} \times 10^{-5})$

The 95% confidence limits of the Eulerian parameter were found to be 0.087 ± 0.016 at $N_{Re} = 225{,}000$.

CONCLUSIONS

1. Relative intensity of turbulence data have been obtained at lower Reynolds numbers, with water as the fluid. These data appear to corroborate results of other investigators who, however, used air as the fluid.

2. Eulerian integral scales of turbulence have been obtained at these lower Reynolds numbers. Such data do not appear to have been previously available.

3. Correlations having some rational basis have been developed which may be used to relate the relative intensity of turbulence, the Eulerian and Lagrangian integral length scales of turbulence, and the eddy diffusivities with the Reynolds number, for fluid flow in the vicinity of the axis of smooth pipes.

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NOTATION

a = radius of pipe, ft.
C = velocity of particles, ft./sec.
D_E = eddy diffusivity, sq.ft./sec.
D_{mol} = molecular diffusivity, sq.ft./sec.
f = fanning friction factor, dimensionless
l = mean free path, ft.

$\left[\frac{\Lambda_E}{2a}\right]\left[\frac{\sqrt{\overline{u^2}}}{\overline{U}_c}\right] \bigg/ \sqrt{\frac{f}{2}}$ = Eulerian parameter, dimensionless

$\left[\frac{\Lambda_L}{2a}\right]\left[\frac{\sqrt{\overline{C^2}}}{\overline{U}_c}\right] \bigg/ \sqrt{\frac{f}{2}}$ = Lagrangian parameter, dimensionless

$\left[\frac{2a\,\overline{U}_{avg}}{\nu}\right] = N_{Re}$ = Reynolds number, dimensionless

$\left[\sqrt{\overline{u^2}}/\overline{U}_c\right]$ = relative intensity of turbulence at the center of the pipe

$R_L(t')$ = Lagrangian correlation coefficient between velocities of a particle at times t and $t + t'$
$R_E(t')$ = Eulerian correlation coefficient between the turbulent velocities at a point in the fluid at times t and $t + t'$
t = time, sec.
t' = time delay, sec.
u = fluctuating turbulent velocity at a point in the fluid, ft./sec.
u^* = $\sqrt{\frac{f}{2}}\ (\overline{U}_{avg})$ = friction velocity ft./sec.
U = main stream velocity, ft./sec.
$\overline{Y^2}$ = mean square displacement of a particle, sq.ft.
T = integral time scale of turbulence, sec.
Λ = integral length scale of turbulence, ft.

Subscripts

L = Lagrangian
E = Eulerian
avg = average over pipe cross section
c = center of pipe

Superscript

bar = average with respect to time or over a number of particles

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